III. **Solution by W. W. LANDIS, A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.; H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; O. S. WESTCOTT, A. M., Sc. D., Principal North Division High School, Chicago, Ill.; CHAS. E. MEYERS, Canton, Ohio; NELSON L. RORAY, Bridgetown, N. J.; CHAS. C. CROSS, Libertytown, Md.; ELMER SCHUYLER, High Bridge, N. J.; J. H. DRUMMOND, LL. D., Portland, Me.; COOPER D. SCHMITT, A. M. University of Tennessee, Knoxville, Tenn.; and the PROPOSER.**

Let x=distance above the water, y=the required maximum.

Then $40-x$ is the hypotenuse and $\sqrt{[(40-x)^2-(20+x)^2]}=2\sqrt{(300-30x)}$ =the base.

$\therefore\ y:x=2\sqrt{(300-30x)}:20+x.$

$\therefore\ y=\dfrac{2x\sqrt{(300-30x)}}{20+x}$=maximum.

Differentiating and reducing we get $(600-90x)(20+x)-2x(300-30x)=0$.

$\therefore\ x^2+60x=400.$

$\therefore\ x=6.0555128$ feet.

MECHANICS.

63. **Proposed by A. H. BELL, Hillsboro, Ill.**

From a horizontal support at a distance of 10 feet apart, a beam 5 feet long and 10 pounds weight is suspended by ropes attached to each end. The ropes are 3 and 5 feet respectively, in length. Required the angles made by the ropes and horizontal support. Also the stress upon each rope.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa., and the PROPOSER.

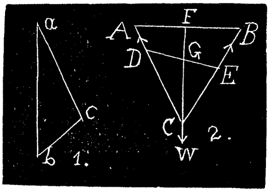
Let $AB=10$, $BC=CD=5$, $AD=3$, $EB=x+y$, $AE=x-y$.

$$\cos AEB=\frac{(x+y)^2+(x-y)^2-100}{2(x+y)(x-y)}=\frac{(x+y-5)^2+(x-y-3)^2-25}{2(x+y-5)(x-y-3)}.$$

$\therefore\ (139-32x)y^2-(200-8x^2)y-79x^2+800x-1500=0$ .... (1).

$$DF=CN=\frac{100+(x-y)^2-(x+y)^2}{20(x-y)}(x-y-3)=\frac{100+(x+y)^2-(x-y)^2}{20(x+y)}(x+y-5).$$

$\therefore\ xy^3-xy^2-(x^3-4x^2+100)y+25x=0$ .... (2).



Let $139-32x=c$, $200-8x^2=2a$,
$79x^2-800x+1500=b$, $x^3-4x^2+100=d$.

$\therefore\ cy^2-2ay-b=0$ .... (3),

and $xy^3-xy^2-dy+25x=0$ .... (4).

y^2 of (3) in (4) $2axy^2+(bx-2ax-cd)y-bx+25cx=0$ .... (5).

$$(5)\times c-(3)\times 2ax,\ y=\frac{bcx-2abx-25c^2x}{bcx+4a^2x-2acx-c^2d}\ldots\ldots\ldots\ldots(6).$$

$$(5)\times b+(3)\times(25cx-bx),\ y=\frac{b^2x-50acx-bcd}{bcx-2abx-25c^2x}\ldots\ldots\ldots\ldots(7).$$

Now $b=25c-79a/4$ and $d=x^3+a$ in $(6)=(7)$, etc.

$$(316a^2x-558acx)^2+(16a^2x-4c^2x^3-4ac^2-87acx+100c^2x)\times$$
$$(6241a^2x+316acx^3+316a^2c-400c^2x^3-400ac^2-16600acx+10000c^2x)=0\ldots.(8).$$

Expanding, we find a common factor c, then by substitution and reduction to the simplest form for the application of Horner's Method.

$$79x^{10}-1748x^9+12559x^8-2429.5x^7-478451.828125x^6$$
$$+2827762.5x^5-4080008.59375x^4-27582812.5x^3$$
$$+161863232.421875x^2-357007812.5x+301890625.=0\ldots.(9).$$

Horner's Method gives $x=7.95690209132$, (3) $y=0.564356664799$.
$x+y=8.521258756118$, $x-y=7.392545426520$.
$CE=3.521258756118$, $DE=4.392545426520$.
$\angle DAB=56°\ 17'\ 54''$, $\angle ABC=46°\ 11'\ 54''$, $\angle AEB=77°\ 30'\ 12''$.
$\angle DCE=59°\ 3'\ 32.5''$, $\angle CDE=43°\ 26'\ 15.5''$.

Tension on $BC=\dfrac{10\cos DAB}{\sin(DAB+ABC)}=5.6863$ pounds.

Tension on $AD=\dfrac{10\cos ABC}{\sin(DAB+ABC)}=7.0896$ pounds.

[NOTE. By a mistake we published the incomplete solution of this problem in our last issue. Soon after receiving that solution, Dr. Zerr wrote us to the effect that a correct and complete solution would shortly follow. Mr. Bell, being ill at the time, was unable to send the complete solution at the time expected, so that by the time the Department was ready for the press, we forgot about the promised complete solution and sent in the incomplete solution for publication. We have not verified the above solution, and our readers must excuse us from that great task. We hold Dr. Zerr and Mr. Bell responsible for any errors contained in it.—ED. F.]

71. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

Three men own a sphere of gold the density of which varies as the square of the distance from the center. If two segments be cut off each one inch from the center of the sphere it will be divided into three parts of equal value. Determine the diameter of the sphere.

II. Solution by R. E. GAINES, A. M., Professor of Mathematics, Richmond College, Richmond, Va.

The element $dydx$ whose ordinate is y will, when revolved about the axis of x generate an infinitesimal ring whose volume is $2\pi ydydx$, and whose distance from the center is $\sqrt{(x^2+y^2)}$. Therefore for the mass of the minor segment we have

$$2\int_0^1\int_0^{\sqrt{(a^2-x^2)}}2\pi y(x^2+y^2)dydx=\pi\int_0^1(a^4-x^4)dx=\pi(a^4-\tfrac{1}{5}).$$